Stray field interaction: isotropic and anisotropic energy in 2D arrays of polarized nano particle

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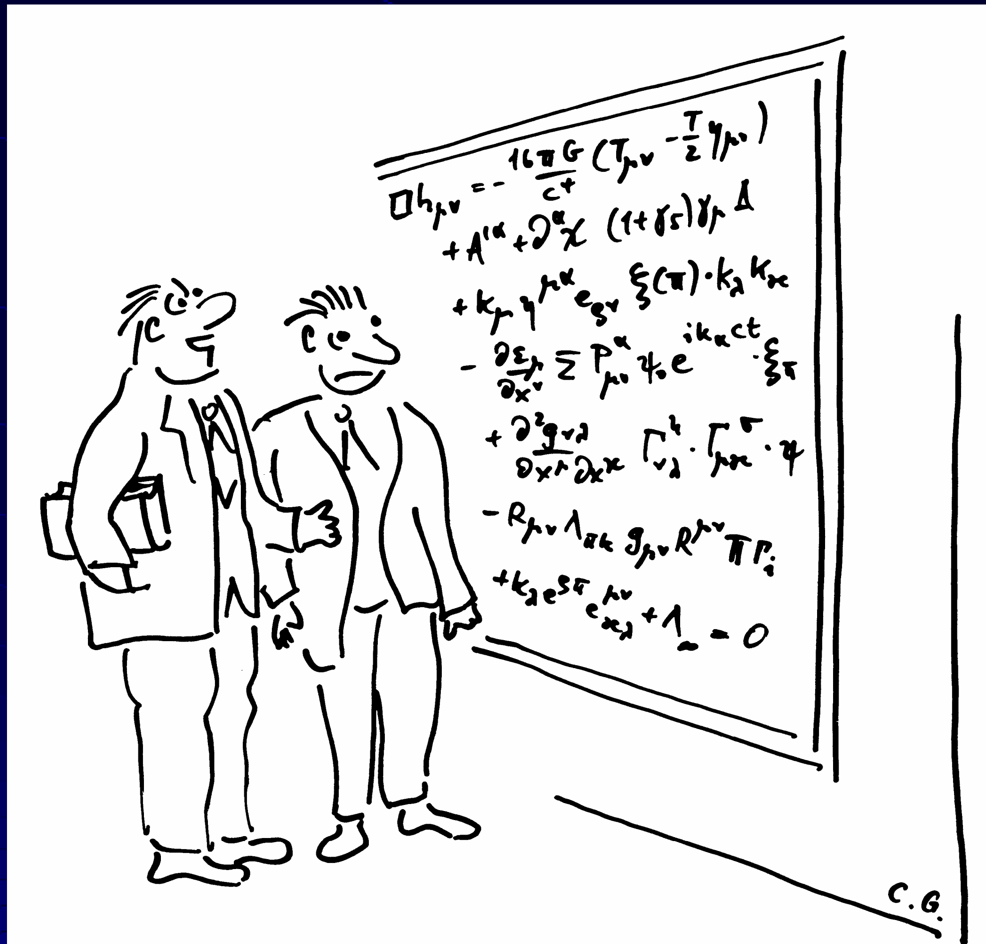
Formulas



This is the simplified version of general relativity… for the students.

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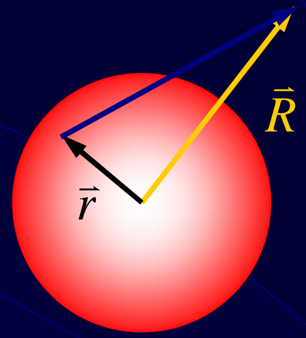
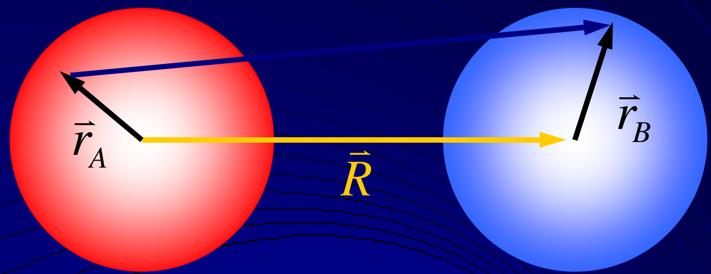
Well known facts

- Multipole expansion describes the potential at “large” distances from the source…



$$\frac{1}{\left|\vec{R}-\vec{r}\right|}$$

- …and the interaction of sources separated by a distance “larger” than their diameter



$$\frac{1}{\left|\vec{R}-\vec{r}_A+\vec{r}_B\right|}$$

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"Well known" facts

- in Cartesian coordinates expansion complexity increases with increasing order

$$\frac{1}{\left|\vec{R}-\vec{r}_A+\vec{r}_B\right|}=\frac{1}{R}\frac{1}{\left|\vec{n}+\frac{\vec{r}_B-\vec{r}_A}{R}\right|}=\frac{1}{R}\frac{1}{\sqrt{1+2\vec{n}\frac{\vec{r}_A-\vec{r}_B}{R}+\frac{(\vec{r}_A-\vec{r}_B)^2}{R^2}}}$$

- complexity is constant for spherical coordinates

$$\frac{1}{\left|\vec{R}-\vec{r}_A+\vec{r}_B\right|}=\sum_{l_Al_Bm_Am_B}T_{l_Al_Bm_Am_B}(\vec{R})R_{l_Am_A}(\vec{r}_A)R_{l_Bm_B}(\vec{r}_B)$$

- many symmetric shapes have analytical solutions for their multipole moments[1]
- a magnetic charge can be defined as

$$\rho=-\mu_0\nabla\vec{M},\sigma=\mu_0\vec{n}\cdot\vec{M}$$

  – frame work of electrostatics sufficient for magnetism

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[1]Mikuszeit, et al. J.Phys. C **16** (2004) 9037
J. Appl. Phys. **97** (2005)103107

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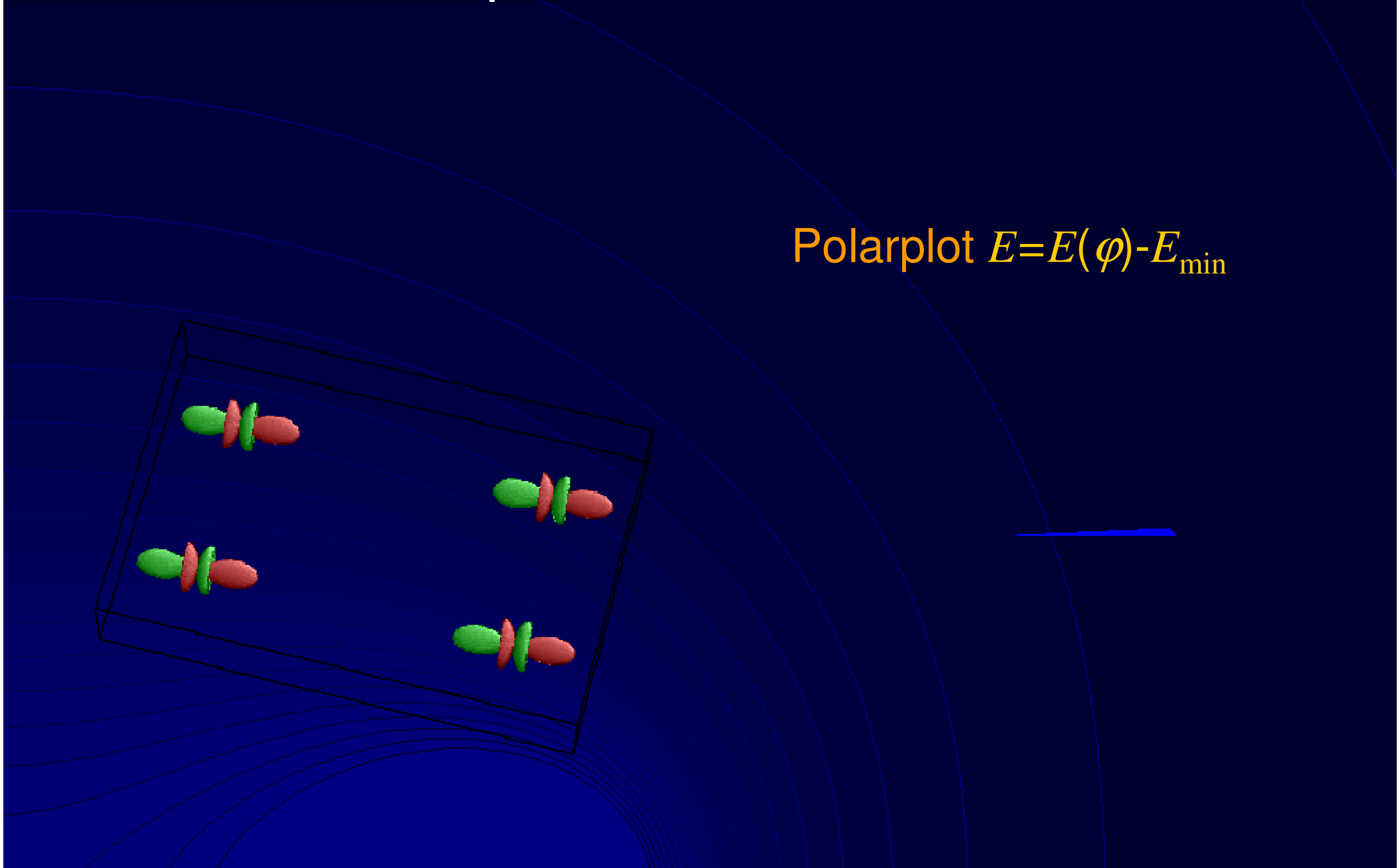
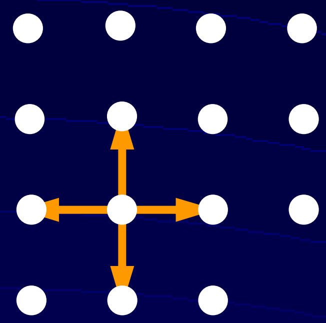
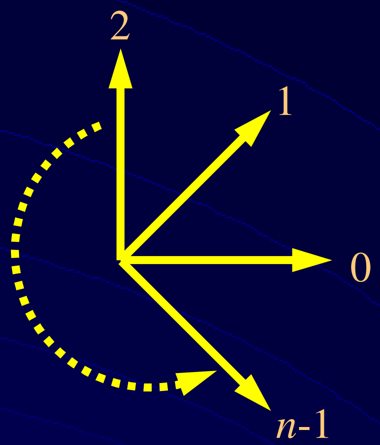
2D lattices, in-plane coherent rotation



It‘s all about symmetry

$$E \propto T_{l_A l_B m_A m_B}(\vec{R}) \propto \frac{Y_{l_A+l_B,-(m_A+m_B)}(\theta,\varphi)}{r^{l_A+l_B+1}} \propto \mathrm{e}^{\mathrm{i}(m_A+m_B)\varphi} = \mathrm{e}^{\mathrm{i}M\varphi}$$

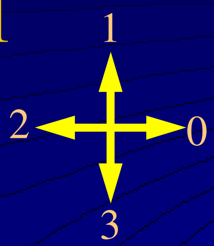




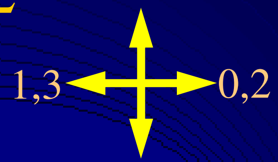
$$E \propto \sum_{k=0}^{n-1} \mathrm{e}^{\mathrm{i}M\varphi_k} = \sum_{k=0}^{n-1} \mathrm{e}^{\mathrm{i}M\,2\pi\frac{k}{n}}$$

Arrow position changes as a function of *M*

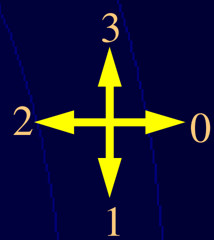
$M = 1$



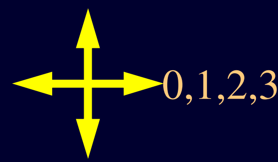
$M = 2$



$M = 3$



$M = \nu n$



$$E \propto n\,\delta_{0,M \bmod (n)}$$

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Further symmetry properties

- Under in-plane rotation multipole moments transform as

$$Q_{lm} \xrightarrow{\gamma} Q_{lm} \mathrm{e}^{\mathrm{i}m\gamma}$$

- Therefore, the product in the interaction transforms propotional to

$$\propto \mathrm{e}^{\mathrm{i}(m_A+m_B)\gamma}$$

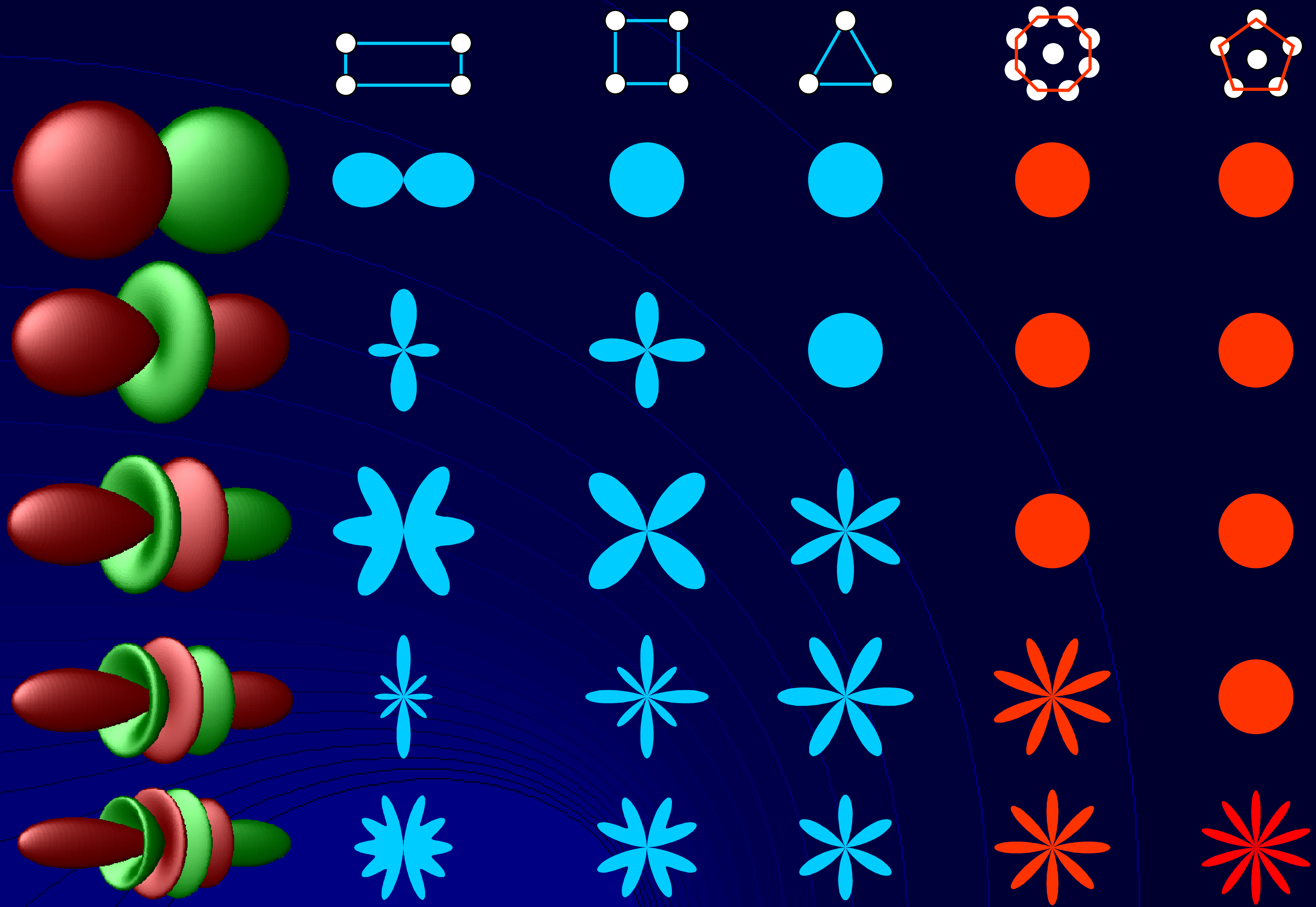
  - where we already know that (m_A+m_B) has to be zero or a multiple of n. The energy than is proportion to

$$\propto \cos(m_A+m_B)\gamma$$

  where $(m_A+m_B)=0$ gives rise to an isotropic energy, while $(m_A+m_B)=k\ n$ result in anisotropic terms

- These properties are even correct for quasi periodic structures with local "disorder", if the energy contribution due to "disorder" cancels to zero on the large scale

Anisotropy surfaces $E - E_{\min} = \sum_{k=1}^{2l} c_k \cos k\varphi$



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Summery

- Any interaction with r^{-1} potential possible
  - electric (molecules, ferroelectic particles)
  - magnetic (nanomagnets, cluster, molecules)
- Symmetry of particles/molecules defines Q_{lm}
- Symmetry n of lattice defines c_k in $E = c_k \cos k\,\gamma$
  - $k=0$ or $k=\nu n$
  - $k \leq L = (l_A + l_B)$
  - on average true in quasi crystals
- Pair interaction and lattice sum are separated
- No energy due to interaction of odd and even order
  - neither isotropic
  - nor anisotropic

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