Maximisation of Stray Field Modulation in Periodic Arrays of Magnetic Particles

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...is important because?

- If periodic arrays of magnetic particles are used for data storage, a proper data read out requires low signal to noise ratio i.e., it is necessary to distinguish two adjacent dots. In the case of completely parallel magnetised dots this is equivalent to a maximisation of the stray field modulation.[1]
- Modulated magnetic fields are interesting in basic research e.g. magneto-transport[2]. The transport properties are modified, which leads to additional effects like Hofstadter butterflies[3]. The stronger the modulation the better.
- Periodic arrays of magnetic particles may also be used for periodic atom-traps in quantum optics[4].

Three voluntary restrictions

The Fourier method allows any direction of magnetisation. Out-of-plane magnetisation leads to the problem of local extrema, as shown later. In-plane magnetisation introduces an additional symmetry breaking. This symmetry breaking drastically increases the number of cases to deal with.

As there is a rich variety of symmetries to treat and in many applications a large modulation of the z-component of the magnetic field is required, we restrict ourselves to perpendicular magnetisation and the maximisation of the z-component of the stray field.

The region of local extrema is not invesigated.

Literature

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[3] D.R. Hofstadter, Phys. Rev. B14, 2239 (1976)
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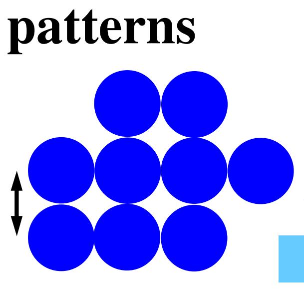
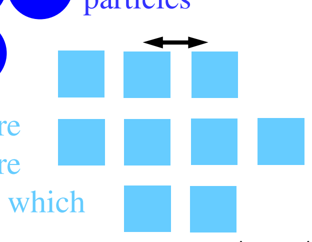
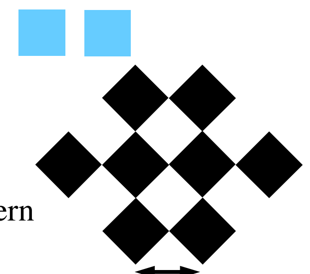
Discussed geometric patterns



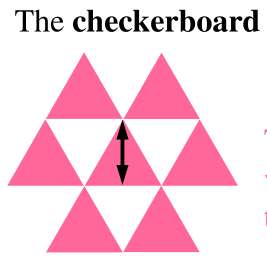
The simple square lattice with disc shaped particles



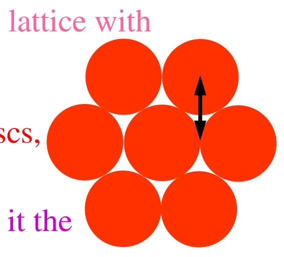
The simple square lattice with square shaped particles, which could also be interpreted as a two fold trench grid.



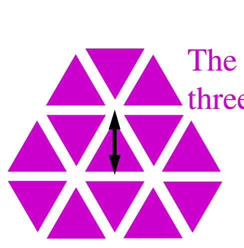
The **checkerboard** pattern



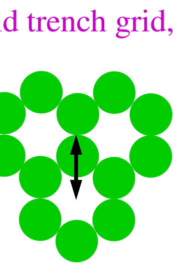
The three fold checkerboard pattern, which is the hexagonal lattice with triangles,



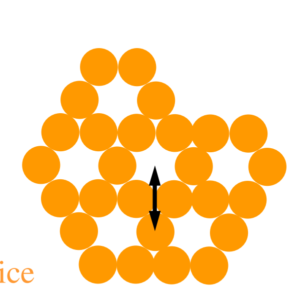
the classical hexagonal (p3) lattice with discs,



The hexagonal (p6), lets call it the three fold trench grid,



the honeycomb lattice and



the Kagome lattice

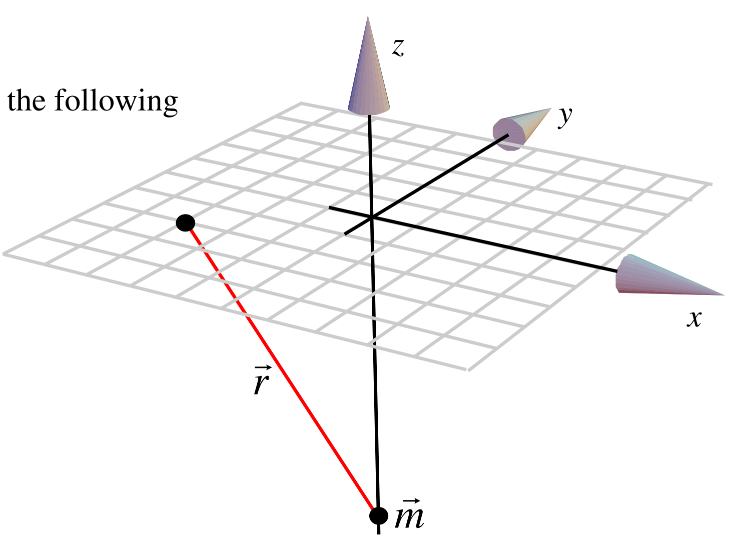
All shown symmetries are scaled to have the same period p, marked by the black arrows. Hence, all symmetries have the same decay length in the far field regime . Apart from the Kagome lattice, which is more of theoretical interest, all symmetries are of technological interest, either due to the fact that the symmetry is simple or they are very easy to produce, e.g. the three fold trench grid by etching or the honeycomb lattice by self organisation.

The Fourier Method[5]

From all possible definitions of the Fourier transform the following is chosen

$$\mathscr{F}(f(x))(k) = \hat{f}(k) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i k\cdot x} dx$$

$$\mathscr{F}^{-1}(\hat{f}(k))(x) = f(x) = \int_{-\infty}^{\infty} \hat{f}(k) e^{2\pi i k\cdot x} dk$$



The potential of a magnetic dipole m is of the form

$$\Phi = -\frac{\vec{m}}{4\pi}\cdot\nabla\frac{1}{\|\vec{r}\|}$$

and its 2D Fourier transform within a plane at distance z is

$$\hat{\Phi}_{\text{dipole}}(k_x,k_y,z) = -\underbrace{\left(\frac{i(k_x m_x + k_y m_y)}{\|\vec{k}\|} + m_z\right)}_{\theta_m}\frac{e^{-2\pi\|\vec{k}\|\cdot|z|}}{2}$$

If the dipole is extended to a particle with height h and lateral shape Ψ, the Fourier transform changes to

$$\hat{\Phi}_{\text{particle}}(k_x,k_y,z) = -\frac{\theta_m}{2}\frac{\sinh \pi h\|\vec{k}\|}{\pi\|\vec{k}\|}e^{-2\pi\|\vec{k}\|\cdot|z|}\cdot\hat{\Psi}$$

In a periodic array the potential is then calculated by the discrete inverse Fourier transform, where k_1, k_2 are the base vectors of the reciprocal lattice.

$$\Phi(x,y,z) = \sum_{\substack{n_1,n_2\in\mathbb{Z}\\ \vec{k}=n_1k_1+n_2k_2}}\hat{\Phi}_{\text{particle}}(k_x,k_y,z)\frac{e^{2\pi i(k_x x + k_y y)}}{V}$$

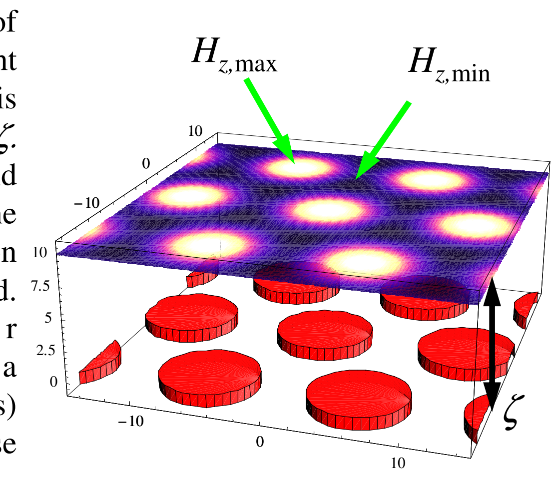
The volume of the real space unit cell is V.

Some interesting properties of the Fourier solution

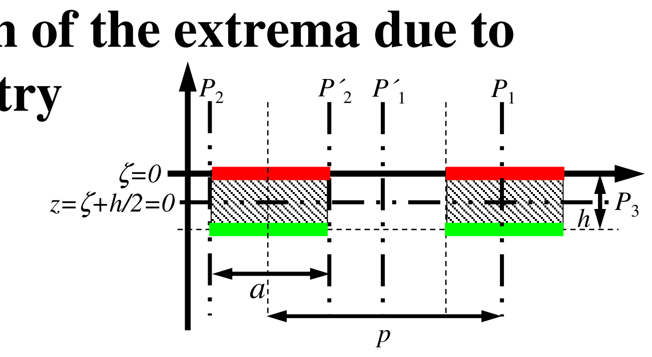
- The Fourier components depend on the distance z.
- The larger z the smaller the influence of high order Fourier components. Consequently, the far field is determined by the smallest Fourier component with the largest k-vector, which corresponds to the periodicity p of the array.
- In the far field the modulation decays exponentially[1] with decay length $p/2\pi$. Therefore, the periodicity defines a critical distance in applications, e.g. for effective storage media readout.
- If z decreases to h the Fourier components diverge, as exp($-2\pi k z$) cannot compensate sinh($\pi h k$).

The system to deal with…

…is an infinite array of particles. The z-component of the magnetic field is calculated at distance ζ. From the extremal field values H_{max} and H_{min} the maximum modulation $\Delta H = H_{max} - H_{min}$ is derived. The particle size (radius r for discs or edgelength a for squares and triangles) is changed to maximise ΔH.



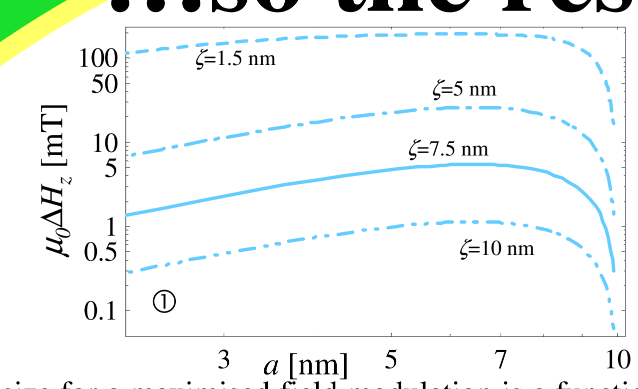
Position of the extrema due to symmetry



Schematic sketch of the system. The particles (hatched rectangles) are magnetised upwards, so on top there is the magnetic north pole (red), at the bottom the south pole (green).

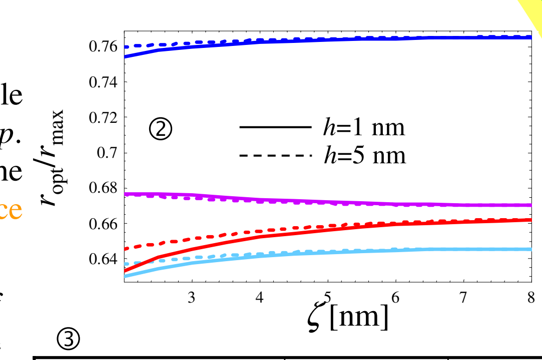
For all symmetries shown below, the particle centre P_1 is a symmetry point. For out-of-plane magnetisation this symmetry is even. Hence, there has to be at least a local extremum at P_1 (only in case of the Kagome lattice a saddle point can form) This extremum is not necessarily the global extremum. The same symmetry considerations are true for P'_1, the symmetry point between particles. In some cases, e.g. the checkerboard pattern, there is an additional symmetry point P_2 with odd symmetry. The z-component of the magnetic field must cross zero at this point.

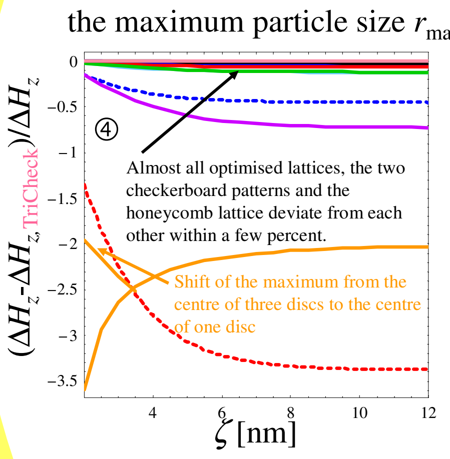
…so the results are…



① The field modulation as a function of particle size shows a maximum (left in case of the two fold trench grid with p=10 nm). The larger ζ, the more narrow the maximum and consequently, the larger the loss in modulation by deviating from the ideal radius.

② The ideal particle size for a maximised field modulation is a function of ζ and the particle height h. The influence of ζ decreases with increasing h. The influence of h vanishes for $h>p$. For $h<p$ the influence of h mainly concerns the formation of local extrema. In case of the **checkerboard pattern**, the three fold checkerboard, the honeycomb and the Kagome lattice the maximum particle size r_{max} is also the ideal one r_{opt}.





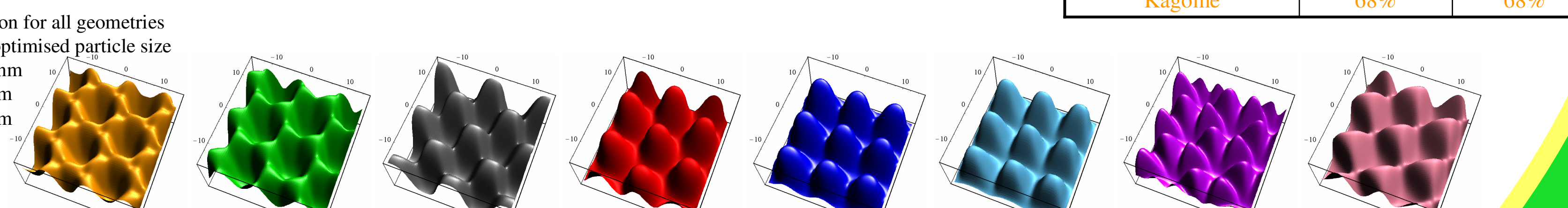
③ Naturally, an ideal fill factor of particles in the array of 50% is expected, but the more complex the symmetry, the stronger the deviation from this value. The table also reveals that the Honeycomb lattice is almost the inverse optimised hexagonal lattice. The Kagome lattice should be seen as a lattice of holes (anti-dot) as well.

④ The **four fold** and three fold checkerboard as well as most of the optimised (continuous lines) lattices give the same modulation. The modulation of the non optimised (dashed lines) hexagonal lattice can be optimised by a factor of 3.4!

③

Lattice/ Shape	max. fill factor	opt. fill factor
Hexagonal/Disc	90.7%	39.8%
Square/ Square	100%	41,1%
Hexagonal/Triangle	100%	44.8%
Square/Disc	78.5%	46.0%
Checkerboard/Square	**50%**	**50%**
Checkerboard/Triangle	50%	50%
Honeycomb	60.5%	60.5%
Kagome	68%	68%

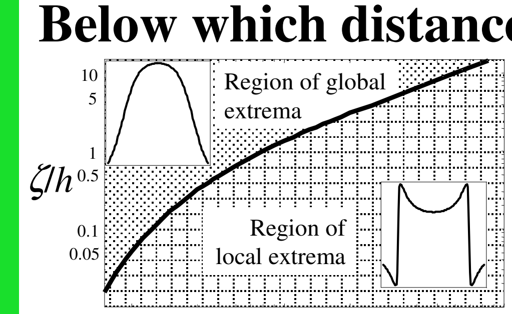
Solution for all geometries with optimised particle size
p=10 nm
h=1 nm
ζ=2 nm



The smallest ζ is specified by limiting technical factors. This defines, due to the exponential decay of the modulation, a critical minimum period p of the array. The checkerboard symmetries always give the best modulation, but other geometries with optimised particle size are comparable.

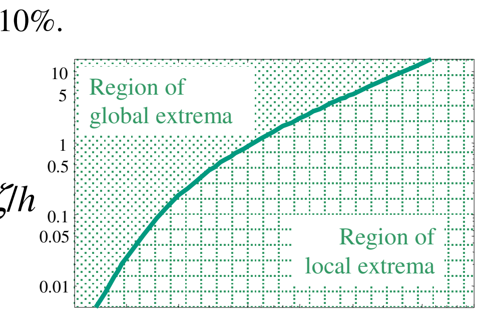
By choosing the correct particle size a strong increase in modulation can be gained!

Below which distance local extrema form?



In case of the checkerboard pattern the local extrema form if ζ is approximately 17% of the period p. That is approximately 25% of the edge length a of the square shaped particles. In case of the Kagome lattice one has approximately ζ/p=10%.

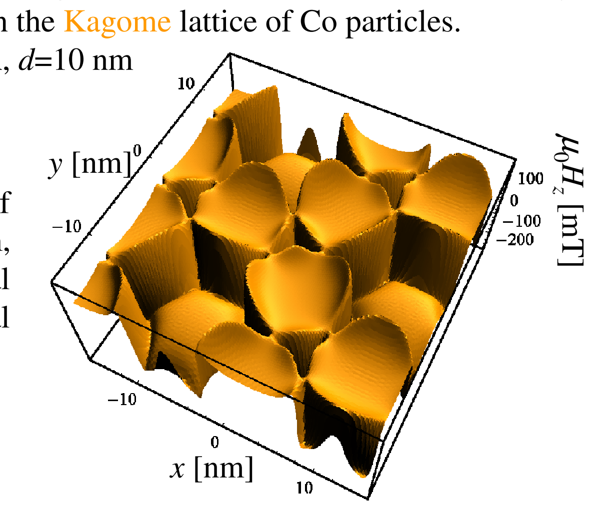
The local minimum above a particle (diameter d, edge length $a=d/\sqrt{2}$) also forms for a single particle and it turns out that it is only slightly modified extending the system to an array. A rule of thumb can be: Local minima form if ζ is less than 25% of the particle diameter.



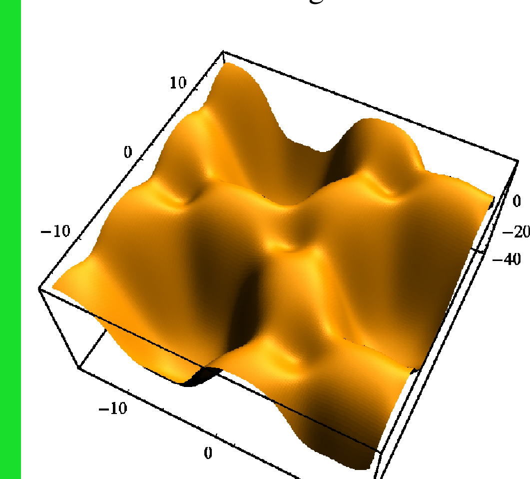
In case of normal memory read out the formation of local extrema, including high frequency components, should be avoided, as the global extrema have a periodicity different from p. In case of read out in terms of edge detection it might even be an advantage.

…so keep in mind that…

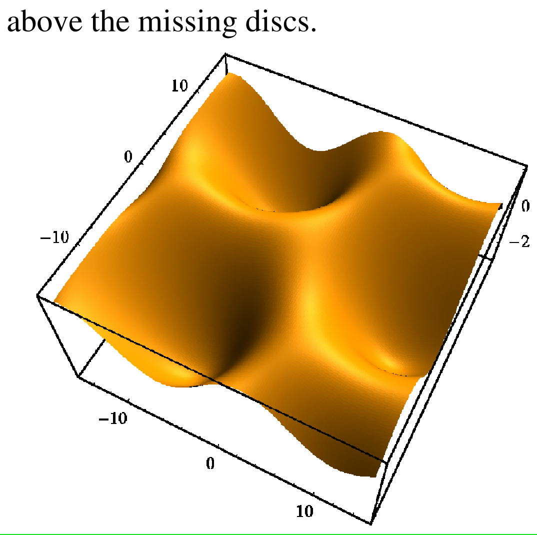
…the position of the global extrema of the magnetic field (z-component) within a plane, coplanar to the array, is not always intuitive and it may change as a function of distance to the array, e.g. in the Kagome lattice of Co particles.
h=5Å, d=10 nm



For a distance **ζ=2.5 Å** the centre of the discs shows a local minimum, above the missing discs a local maximum is observed. The global extrema are at the edges of the discs.



For **ζ=3 nm** the global maxima are above the centre of the discs, the global minima are above the missing discs.



For **ζ=10 nm** the global maxima shifted above the centre of three adjacent discs. Above the discs a saddle point has formed.